

International Journal of Modern Physics E
 © World Scientific Publishing Company

AN UPDATE OF MUON CAPTURE ON HYDROGEN

SAORI PASTORE*

*Department of Physics and Astronomy, University of South Carolina
 Columbia, SC 29208, USA*

FRED MYHRER†

*Department of Physics and Astronomy, University of South Carolina
 Columbia, SC 29208, USA*

KUNIHARU KUBODERA‡

*Department of Physics and Astronomy, University of South Carolina
 Columbia, SC 29208, USA*

Received Day Month Year
 Revised Day Month Year

The successful precision measurement of the rate of muon capture on a proton by the MuCap Collaboration allows for a stringent test of the current theoretical understanding of this process. Chiral perturbation theory, which is a low-energy effective field theory that preserves the symmetries and the pattern of symmetry breaking in the underlying theory of QCD, offers a systematic framework for describing μp capture and provides a basic test of QCD at the hadronic level. We describe how this effective theory with no free parameters reproduces the measured capture rate. A recent study has addressed new sources of uncertainties that were not considered in the previous works, and we review to what extent these uncertainties are now under control. Finally, the rationale for studying muon capture on the deuteron and some recent theoretical developments regarding this process are discussed.

Keywords: electroweak nucleon form factors; chiral effective field theory.

PACS numbers: 12.39.Fe, 13.40.Ks, 25.30.Mr, 23.40.-s

1. Introduction

The highly precise measurement of the $\mu^- p$ capture rate provides us with stringent constraints on our theoretical understanding of QCD at work in hadrons. The $\mu^- p$ capture occurs primarily from the hyperfine-singlet state of a muonic hydrogen atom.¹ The hyperfine-singlet capture rate Γ_0 has recently been measured by the

*pastores@mailbox.sc.edu

†myhrer@physics.sc.edu

‡kubodera@physics.sc.edu

2 *Saori Pastore, Kuniharu Kubodera, Fred Myhrer*

MuCap Collaboration² with very high accuracy ($\sim 1\%$ precision); the reported value is

$$\Gamma_0^{\text{exp}}(\mu^- p \rightarrow \nu_\mu n) = 714.9 \pm 5.4(\text{stat}) \pm 5.1(\text{syst}) \text{ s}^{-1}. \quad (1)$$

Moreover, an ongoing experiment by the MuSun Group³ envisages to measure, with 1.5% precision, the $\mu^- d$ capture rate from the hyperfine-doublet state of a $\mu - d$ atom, while the $\mu - {}^3\text{He}$ capture rate has been already measured with 0.3% precision.⁴

The recent years have witnessed a significant advancement in the theoretical framework of heavy-baryon chiral perturbation theory (HB χ PT), a low-energy effective field theory (EFT) of QCD. One of the remarkable features of HB χ PT is that it allows us to systematically describe electroweak processes involving the nucleon and light nuclei. The main goal of this review article is to survey the latest theoretical progress that has close bearing upon the above-mentioned experimental developments concerning muon capture on nucleons and the lightest nuclei. This article is not intended to be a comprehensive review of muon capture in general, and for the topics that are not covered here, we refer the reader to the recent review articles of Refs.^{5–8}

We give in Sec. 2 a highly abridged recapitulation of HB χ PT, just to provide terms and define notations needed for this review. In Sec. 3, we discuss the pseudo-scalar form factor that appears in the matrix element of the axial-vector current for the nucleon. The importance of radiative corrections along with their latest evaluations are also discussed. In Sec. 4, we present the current status of theoretical calculations of the $\mu^- p$ capture rate. Sec. 5 is devoted to a general discussion on two-nucleon electroweak processes. The latest calculations of the $\mu^- d$ capture rate are reported in Sec. 6, while discussion and a summary are provided in Sec. 7.

2. Heavy-Baryon Chiral Perturbation Theory

In describing low energy-momentum hadronic phenomena characterized by a scale Q that is sufficiently small compared with the chiral scale $\Lambda_\chi \sim 1 \text{ GeV}$, we can eliminate from the Lagrangian those degrees of freedom that pertain to scales higher than Λ_χ . The resulting EFT, called chiral perturbation theory (χ PT), is a low-energy EFT of QCD.⁹ The χ PT Lagrangian, $\mathcal{L}_{\chi\text{PT}}$, contains as explicit degrees of freedom only those hadrons that have masses significantly lower than Λ_χ , and the terms in $\mathcal{L}_{\chi\text{PT}}$ are organized into a perturbative expansion in powers of $\epsilon = Q/\Lambda_\chi \ll 1$. By construction, $\mathcal{L}_{\chi\text{PT}}$ retains all symmetries of QCD, including (approximate) chiral symmetry. The effective nature of the theory is reflected in the presence of low-energy constants (LECs), which parametrize the high-energy dynamics that has been eliminated (integrated out) in generating the low-energy EFT. If the quarks are massless, the QCD Lagrangian is chirally symmetric. This symmetry is spontaneously broken, leading to the existence of massless pseudo-scalar bosons, *i.e.*, the Nambu-Goldstone bosons. In the non-strange sector of our concern here,

the Nambu-Goldstone bosons are massless pions. Chiral symmetry is also explicitly broken by non-zero u and d quark masses which cause the pion to acquire a finite mass, m_π . Since $m_\pi \ll \Lambda_\chi$, the explicit chiral symmetry breaking effect can be accounted for through an additional expansion in the small parameter m_π/Λ_χ . The latter is implicit in the expansion parameter defined above, that is $\epsilon = Q/\Lambda_\chi$, where now Q denotes either the typical size of the four-momentum involved in the process under consideration or the pion mass.

After the successful application to the meson sector,^{10,11} χ PT has been extended to study processes that involve nucleons. In the low-energy regime of interest here, it is reasonable to treat nucleons as non-relativistic particles, and accordingly we suppress antinucleon degrees of freedom and retain only the “large” components of the nucleon field. The resulting theory is HB χ PT which involves an expansion parameter $\epsilon' = Q/m_N$ (where m_N is the nucleon mass) in addition to the ϵ parameter defined above. Since $\Lambda_\chi \approx m_N$, it is a common practice to combine the expansions in ϵ and ϵ' ; thus, n -th order terms in HB χ PT are those terms with a combined power of ϵ and ϵ' equal to n . For review articles, we refer to, *e.g.*, Refs.^{12–14}

The LECs contained in the HB χ PT Lagrangian $\mathcal{L}_{\text{HB}\chi\text{PT}}$ can in principle be determined from lattice QCD calculations, but in practice they are fixed by fitting appropriate experimental data. Once all the LECs at a given order in the expansion are determined, HB χ PT allows us to make model-independent predictions (to that order) on observables other than those used to fix the LECs.

3. Nucleon Pseudoscalar Form Factor

Weak processes, occurring at energies which are very small compared to the weak bosons masses, can be described with high accuracy by the Fermi current-current interaction. In particular, the weak Hamiltonian, relevant to the $\mu^- + p \rightarrow n + \nu_\mu$ reaction, is given by the product of the leptonic (L_μ) and hadronic (J_μ) currents, as

$$H_{\text{weak}} = \frac{G_F}{\sqrt{2}} V_{ud} L_\mu J^\mu, \quad (2)$$

where $G_F = 1.16637(5) \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant while $V_{ud} = 0.97418(27)^{15}$ is the CKM (Cabibbo-Kobayashi-Maskawa) matrix element. The leptonic current is simply $L_\mu = \bar{\psi}_{\nu_\mu} \gamma_\mu (1 - \gamma_5) \psi_\mu$, where ψ_{ν_μ} (ψ_μ) is the neutrino (muon) wave function. By contrast, the hadronic current J^μ does not have a simple form due to complications induced by the strong interactions. We can however parametrize the possible form of its matrix element for a case in which the initial and final states are nucleons. Thus, for $J_\mu = V_\mu - A_\mu$, where V_μ and A_μ are the vector and axial-vector currents, respectively, we can write

$$\langle n(p') | V_\mu | p(p) \rangle = \bar{u}_n(p') \left[F_V(q^2) \gamma_\mu + \frac{i F_M(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu \right] u_p(p), \quad (3)$$

$$\langle n(p') | A_\mu | p(p) \rangle = \bar{u}_n(p') \left[G_A(q^2) \gamma_\mu \gamma_5 + G_P(q^2) \frac{q_\mu}{m_\mu} \gamma_5 \right] u_p(p), \quad (4)$$

4 *Saori Pastore, Kuniharu Kubodera, Fred Myhrer*

where $q = p' - p$ is the momentum transfer with p (p') being the proton (neutron) momentum; $m_N = (m_p + m_n)/2$ is the average nucleon mass, and m_μ the muon mass. The $F_V(q^2)$, $F_M(q^2)$, $G_A(q^2)$ and $G_P(q^2)$ are called the vector, weak-magnetism, axial-vector and pseudo-scalar form factors, respectively, and they account for the composite structure of the nucleons. In the above expressions, we have ignored possible contributions from second-class currents.¹⁶ The μ^-p capture reaction is the most suited process for obtaining information on the pseudoscalar form factor $G_P(q^2)$.¹ Bernard *et al.*¹⁷ derived $G_P(q^2)$ using HB χ PT at one-loop order and obtained

$$G_P(q^2) = \frac{2m_\mu g_{\pi NN} f_\pi}{m_\pi^2 - q^2} - \frac{1}{3} g_A m_\mu m_N \langle r_A^2 \rangle, \quad (5)$$

where $g_{\pi NN}$ is the strong pion-nucleon coupling constant, and f_π is the pion decay constant. The leading term in this expression is the well-known pion-pole term,¹⁷ while the second term involves the nucleon's mean-square isovector axial-radius, $\langle r_A^2 \rangle$, which is related to the axial form factor via $G_A(q^2) = G_A(0) [1 + \frac{1}{6} \langle r_A^2 \rangle q^2 + \dots]$. More recently, Fearing *et al.*¹⁸ also derived Eq. (5) in a slightly different HB χ PT formulation. Historically, the result given in Eq. (5) was obtained in the late sixties/early seventies by Adler and Dothan¹⁹ using the soft-pion theorems, and by Wolfenstein²⁰ using dispersion theory.

A great merit of HB χ PT is that it allows us to estimate the size of errors associated with a given theoretical calculation. In the case of the nucleonic pseudoscalar form factor, corrections at two-loop order have been explicitly evaluated by Kaiser,²¹ and found to be negligible, provided that the involved LECs were of natural size. When we insert in Eq. (5) the momentum transfer pertaining to the μ^-p capture reaction, *i.e.* $q^2 = -0.88m_\mu^2$, along with the experimentally determined axial radius²² $\langle r_A^2 \rangle = 0.44 \pm 0.02 \text{ fm}^2$, HB χ PT at one-loop order gives $G_P(q^2 = -0.88m_\mu^2) = 8.26 \pm 0.23$, which is in excellent agreement with the empirical value of 8.06 ± 0.55 , obtained by the recent MuCap experiment.² The details of the framework used in obtaining this experimental value was thoroughly reviewed in Ref.⁵ It should be stressed that, in order to match the 1% accuracy achieved in the measurement of the μ^-p capture rate, radiative corrections need to be carefully taken into account; the MuCap group used the radiative corrections evaluated by Czarnecki *et al.*²³ Since the time when Ref.⁵ was written, there have been significant developments which affect the theoretical description of the μ^-p capture reaction, and these developments are reviewed in the next section.

4. The μ^-p Capture Rate

The 1% experimental accuracy achieved by the MuCap Collaboration² in the measurement of Γ_0 , poses a challenge for the theory. To attain a comparable theoretical precision, higher-order HB χ PT contributions, including radiative corrections, need to be accounted for. In HB χ PT the μ^-p capture rate has been evaluated by Fearing *et al.*,¹⁸ Ando *et al.*,²⁴ and Bernard *et al.*²⁵ In these works the transition amplitude

was evaluated including m_N^{-1} nucleon recoil corrections entering at next-to-leading order (NLO). At next-to-next-to-leading order (N²LO), there are recoil corrections of order m_N^{-2} as well as loop corrections. Since all the LECs at N²LO are known, HB χ PT leads to model-independent predictions for the μ^-p capture rate. Based on the convergence pattern exhibited by the contributions to the capture rate evaluated in, *e.g.*, Ref.,²⁵ it is estimated that N³LO corrections would contribute at the 1% level to the capture rate. Comparison of the results for Γ_0 obtained in HB χ PT with the earlier results obtained in the phenomenological approach, *e.g.*, Refs.^{1,26} can be found in Refs.^{5,6}

A recent HB χ PT calculation of μ^-p capture²⁷ takes into account radiative corrections of order $\alpha \sim 1/137$, which enter at N²LO in the chiral expansion, that is, they scale as $(Q/\Lambda_\chi)^2$; the fact that $Q \sim m_\mu$ in μ^-p capture leads to the relation $(Q/\Lambda_\chi)^2 \sim (m_\mu/\Lambda_\chi)^2 \sim 1/100 \sim \alpha$. These radiative corrections include standard QED vacuum polarization effects,²⁸ electroweak loop corrections, as well as proton finite-size corrections.²⁹ Divergences generated by electroweak loops appearing at N²LO are regulated by electroweak LECs, which describe short-distance effects. These LECs represent the “inner” corrections in the formalism of Sirlin,³⁰ and are determined by matching the expressions for the neutron β -decay radiative corrections obtained by Marciano and Sirlin,³¹ and those derived in HB χ PT by Ando *et al.*³² The radiative corrections derived in Ref.²⁷ are found to be in agreement with those evaluated by Czarnecki *et al.*,²³ which have been used by the MuCap Collaboration. In Ref.,²⁷ it was also found that electroweak loop-corrections increase the calculated rate Γ_0 by as much as $\sim 2\%$, an increase that, due to partial cancellations among other terms,²⁷ is dominated by the aforementioned electroweak LECs. In addition, Raha *et al.*²⁷ showed that, even if we generously assign a 10% uncertainty to the nucleon isovector axial radius, $\langle r_A^2 \rangle^{1/2}$, the corresponding variation in Γ_0 is within $\sim 0.5\%$.

Apart from the above-mentioned $\sim 1\%$ uncertainty due to N³LO contributions, the N²LO calculation of Γ_0 involves additional uncertainties. These arise from uncertainties associated with the nucleon axial-vector coupling constant, g_A , and the nucleon-pion coupling constant, $g_{\pi NN}$. The axial constant g_A is determined most directly from the measured asymmetry parameter A in neutron beta decay.^{33,34} Historically, the value of g_A recommended by the Particle Data Group (PDG) has steadily increased, and the latest reported value is $g_A = 1.2701 \pm 0.0025$.¹⁵ Unfortunately, this is not the last word in the saga of g_A . The asymmetry parameter A in neutron beta decay has recently been re-measured by two groups,^{33,34} and they have obtained $g_A \simeq 1.276$, which is larger than the PDG2012 value.¹⁵ It should be noted that the value $g_A \simeq 1.276$ is more consistent with the smaller value of the neutron life time, $\tau_n = 880.0 \pm 0.9$ s, which is now recommended by the PDG;¹⁵ see the arguments in Ref.³⁵ advocating for a smaller value of τ_n . The relation between the new larger value of g_A and the smaller τ_n has also been discussed in Refs.^{33,36} We note that the value of the neutron lifetime is not settled experimentally, as shown

6 *Saori Pastore, Kuniharu Kubodera, Fred Myhrer*

Table 1. Variations of the μ^-p capture rate Γ_0 in s^{-1} and the Goldberger-Treiman discrepancy, Δ_{GT} , are given with respect to some selected values for g_A and $g_{\pi NN}$. The radiative corrections discussed in Ref.²⁷ are accounted for.

g_A	$g_{\pi NN}$	Δ_{GT}	Γ_0
1.266	13.40	-0.040	707.1
1.2761	13.40	-0.036	715.8
1.266	13.044	-0.014	710.4
1.2761	13.044	-0.006	719.2

by Yue *et al.*³⁷ The pion-nucleon coupling constant $g_{\pi NN}$ has been extracted from both nucleon-nucleon and pion-nucleon scattering data, as discussed recently in, *e.g.*, Refs.^{38–43} No consensus has been reached on the value of $g_{\pi NN}$, and the best we can do at present is to allow $g_{\pi NN}$ to have a range, $g_{\pi NN} = 13.044\text{--}13.40$; the smaller value is taken from Ref.⁴⁴ and the larger one from Ref.⁴⁵ The uncertainty in $g_{\pi NN}$ affects the evaluation of Γ_0 at N²LO via the Goldberger-Treiman discrepancy, $\Delta_{GT} = g_A m_N / (g_{\pi NN} f_\pi) - 1$.

Given the changing value of g_A and the existing uncertainty in $g_{\pi NN}$, it is important to estimate variations in Γ_0 due to changes in g_A and $g_{\pi NN}$. Such an estimation has been carried out by Pastore *et al.*,⁴⁶ and their results are shown in Table 1. Note that all the theoretical values for Γ_0 in Table 1 are within the experimental errors given in Eq. (1). If we use the latest published values for g_A and $g_{\pi NN}$,^{33,42,43} the larger Γ_0 in the last row appears theoretically favored. Variations in the calculated value of Γ_0 due to the existing uncertainties in g_A and $g_{\pi NN}$ are of comparable size to the estimated contributions from N³LO terms.^{24,25,46} Therefore, it does not seem warranted at present to go on to N³LO calculations, which involve a major effort.

5. Family of Two-Nucleon Weak-Interaction Processes

There exists a long list of literature on the evaluation of the μ^-d capture rate;^{47–50} the most recent works are strongly motivated by the ongoing experimental effort by the MuSun collaboration³ at PSI, which aims at measuring it at 1.5% precision. In the recent theoretical developments, HB χ PT has been playing an important role, as described below.

The extension of HB χ PT to multi-nucleon systems is accomplished following the scheme formulated by Weinberg in Refs.^{51–54} The basic idea is to categorize Feynman diagrams describing a given reaction into irreducible and reducible diagrams. Irreducible diagrams are those that do not involve pure nucleonic intermediate states, and all other diagrams are called reducible. Let us consider a two-nucleon system as an example. The HB χ PT nucleon-nucleon potential, v_{ij}^{EFT} , is defined as the sum of all the irreducible diagrams entering the $NN \rightarrow NN$ transitions am-

plitude. The contributions of reducible diagrams can be included by solving the Schrödinger equation in which v_{ij}^{EFT} appears as the potential. The HB χ PT three-nucleon potential, v_{ijk}^{EFT} , can be defined in a similar manner. For an A -body system, the nuclear wave function Φ^{EFT} is a solution of the A -body Schrödinger equation with the Hamiltonian given by

$$H^{\text{EFT}} = \sum_{i=1}^A K_i + \sum_{i<j}^A v_{ij}^{\text{EFT}} + \sum_{i<j<k}^A v_{ijk}^{\text{EFT}} + \dots, \quad (6)$$

where K_i is the kinetic energy of the i th nucleon; the dots denote operators involving more than three nucleons, which are of higher order in the HB χ PT expansion and hence can be dropped. The matrix element of a nuclear electroweak transition is given by

$$\mathcal{M}^{\text{EFT}} = \langle \Phi_f^{\text{EFT}} | \sum_i^A \mathcal{O}_i^{\text{EFT}} + \sum_{i<j}^A \mathcal{O}_{ij}^{\text{EFT}} + \dots | \Phi_i^{\text{EFT}} \rangle, \quad (7)$$

where the initial and final wave functions are obtained in the manner described above. The transition operators can have terms involving three or more nucleons, but they are of higher orders in the HB χ PT expansion. The one-body (two-body) transition operator, $\mathcal{O}_i^{\text{EFT}}$ ($\mathcal{O}_{ij}^{\text{EFT}}$), is obtained as the sum of all irreducible diagrams involving the relevant external current for one-nucleon (two-nucleon) diagrams. The derivation of these operators in HB χ PT was pioneered by Park, Min, and Rho in Ref.⁵⁵ for the electromagnetic current, and in Ref.⁵⁶ for the weak axial current; $\mathcal{O}_i^{\text{EFT}}$ and $\mathcal{O}_{ij}^{\text{EFT}}$ were derived up to N³LO. At this order the two-body operators $\mathcal{O}_{ij}^{\text{EFT}}$ include contributions from one- and two-pion exchanges. More recently, chiral electromagnetic current (and charge) operators at one-loop order have been derived by Kölling *et al.* in the unitary transformation method,^{57,58} and by Pastore *et al.* within time-ordered perturbation theory.^{59–61} These two approaches differ among themselves and from the scheme adopted by Park *et al.*, in the treatment of the reducible contributions. A discussion on these differences can be found in Refs.^{58–61} A derivation of the axial current within the formalism developed in Refs.^{59–61} is being vigorously pursued.⁶²

In considering the specific case of μ^-d capture, we note the following two crucial points: (i) for the low-energy Gamow-Teller (GT) transition which governs this process, the one-body transition operator, $\mathcal{O}_i^{\text{EFT}}$, is well known, see Eq. (4); (ii) the two-body terms, $\mathcal{O}_{ij}^{\text{EFT}}$, involve only one unknown LEC, which in the literature is denoted by d^R . This LEC parameterizes the strength of a contact-type four-nucleon coupling to the axial current; diagram d) in Fig. 1 illustrates this coupling. Thus d^R can be regarded as the two-nucleon analog of the nucleon axial-vector coupling constant g_A .

As noted by Park *et al.*,⁶³ d^R also enters the two-body GT amplitude of the solar pp fusion reaction, tritium β -decay,^{63,64} and νd scattering at low energies.⁶⁵ This means that, if d^R can be determined from the experimentally known rate of any

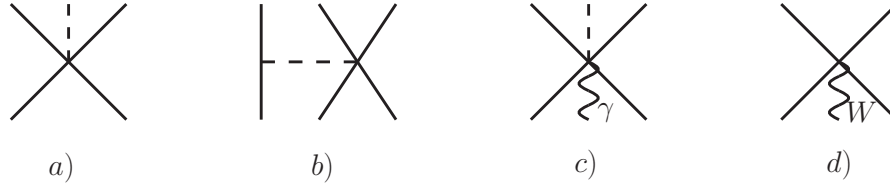
8 *Saori Pastore, Kuniharu Kubodera, Fred Myhrer*

Fig. 1. Diagrams involving the LEC, d^R . The solid and dashed lines represent nucleons and pions, respectively. The wavy line in diagram c) [diagram d)] represents a photon (W weak boson).

one of these processes, robust predictions can be made for the remaining reactions. Moreover, d^R enters pure hadronic as well as electromagnetic reactions. To the former class belong, for example, the processes represented by diagrams a) and b) in Fig. 1. Diagram a) appears in the hadronic reaction $NN \rightarrow NN\pi$.^{66,67} Diagram b) contributes to three-nucleon interactions, giving rise to a relation between d^R and c_D , an LEC that parameterizes the short-range contribution to the three-nucleon potential.^{68–71} Diagram c) represents an electromagnetic process that involves d^R . This diagram appears in, *e.g.*, the $\gamma d \rightarrow \pi NN$ reaction^{72,73} and $\pi^- d \rightarrow \gamma NN$ reaction.^{74,75} The last reaction has long been known as a tool to extract the nn -scattering length, and a detailed HB χ PT study of this extraction procedure has recently been made by Gardestig and Phillips.^{76,77}

In all the reactions given above, the short-ranged operator accompanied by the LEC d^R parameterizes common short-distance two-nucleon physics that has been integrated out. How these processes are interconnected can be easily understood by examining the structure of the chiral Lagrangian, which is customarily written in terms of the chiral field $U(x)$.^a The contact interaction, illustrated by the diagrams of Fig. 1, is given by a four-nucleon interaction Lagrangian of the form

$$\mathcal{L}_{NN} = -2d (N^\dagger S \cdot u N) N^\dagger N, \quad (8)$$

where $N(x)$ is the heavy-nucleon field, S^μ is the nucleon covariant spin operator, and $u_\mu \equiv i(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$, with $\xi = \sqrt{U(x)}$. The coupling constant d becomes d^R after the renormalization procedure is implemented. Including the external electroweak currents, V_μ and A_μ , we can see that u^μ connects the pion emission vertex with the external vector and axial-vector currents via

$$f_\pi u_\mu = -\tau \partial_\mu \pi - \varepsilon_{3ab} V_\mu \pi_a \tau_b + f_\pi A_\mu + \dots, \quad (9)$$

where the ellipses represent higher powers in the pion field. The contributions of the first term in Eq. (9) to the contact Lagrangian, \mathcal{L}_{NN} , give rise to the vertices appearing in diagrams a) and b) in Fig. 1, while the second and third terms in Eq. (9) generate the vertices appearing in diagrams c) and d).

^a Ref.⁵⁶ defines $U(x) = \exp[i\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)/f_\pi]$, whereas Ref.¹² uses the “sigma-gauge” expression $U(x) = \sqrt{1 - (\boldsymbol{\pi}/f_\pi)^2} + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)/f_\pi$.

The first determination of the LEC d^R from experimental data was done in Ref.⁶³ by reproducing the tritium β -decay rate, Γ_β^t . In a recent work Gazit *et al.*⁶⁸ used the ^3H and ^3He binding energies as well as Γ_β^t to fix d^R . Although there are good reasons to believe that the determination of d^R with the use of observables in the $A = 3$ systems is reliable to the quoted level, it is desirable to determine d^R within the two-nucleon system without resorting to the input from the three-nucleon system. As discussed in the next section, the high-precision measurement of the μ^-d capture rate by the MuSun group³ is of particular importance for the determination of d^R .

6. Muon-Deuteron Capture Rate

Recent experimental and theoretical developments have reached such a level of accuracy that all the relevant LECs are controlled with reasonable precision. Meanwhile, HB χ PT studies of the two-nucleon systems have established that the low-energy weak-interaction processes in the $A=2$ systems involve only one unknown LEC, d^R , up to N³LO.⁵⁶ This means that if we can carry out an explicit calculation of \mathcal{M}^{EFT} in Eq.(7), and if sufficiently accurate data on μ^-d capture becomes available, then d^R can be fixed. This will allow us to correlate in a reliable model-independent manner, all the low-energy electroweak processes in the two-nucleon systems. The on-going high-precision measurement of the μ^-d -capture rate by the MuSun Collaboration is expected to play an important role in this program; cf. *e.g.*, Refs.^{3,47–50}

To set the stage for surveying the recent developments based on HB χ PT, we first briefly describe the traditional method known as the standard nuclear physics approach (SNPA). SNPA starts with the assumption that an A -nucleon system is described by the Hamiltonian

$$H^{\text{SNPA}} = \sum_{i=1}^A K_i + \sum_{i<j}^A v_{ij}^{\text{SNPA}} + \sum_{i<j<k}^A v_{ijk}^{\text{SNPA}}, \quad (10)$$

where v_{ij}^{SNPA} (v_{ijk}^{SNPA}) is a high precision phenomenological two-body (three-body) potential. These potentials are constrained by reproducing existing two-nucleon scattering data as well as the binding energies and level structure of light nuclei, etc.; see, *e.g.*, Refs.^{78,79} The electroweak transition operators in SNPA consist of one-body impulse-approximation (IA) terms, and two-body meson exchange-current (MEC) terms; the IA terms can be determined from the coupling of a single nucleon to the electroweak current, while the MEC terms are derived from boson-exchange diagrams. SNPA has been applied with great success to the description of nuclear observables in light nuclei, see, *e.g.*, Ref.⁸⁰ Detailed calculations of μ^-d capture based on SNPA were carried out by Tatara *et al.*⁸¹ and by Adam *et al.*⁸² more than twenty years ago. Tatara *et al.* obtained for the hyperfine-doublet capture rate $\Gamma_d(\text{SNPA}) = 300 - 400 \text{ s}^{-1}$, and it was noted that more than 50 % of the contributions to Γ_d come from higher partial-wave states ($L \geq 1$) for the final two-nucleon relative motion. Even though SNPA is believed to work with reasonable

accuracy, it involves a certain degree of model dependence. In principle, HB χ PT should allow us to treat multi-nucleon systems in a model-independent way.

In the past it was a challenge to generate, strictly within the EFT framework, nuclear wave functions with accuracy comparable to that of the SNPA nuclear wave functions. To avoid this difficulty, Park *et al.*⁶³ proposed to replace Φ^{EFT} in Eq.(7) with Φ^{SNPA} , where Φ^{SNPA} is a phenomenological nuclear wave function obtained as an exact eigenstate of the nuclear Hamiltonian H^{SNPA} in Eq. (10). This hybrid method, termed EFT*, has the advantage that it can be applied to complex nuclei ($A = 3, 4, \dots$) with essentially the same precision as to the $A = 2$ case; it thus allows us to fix d^R from observables pertaining to complex nuclei as was done in, *e.g.*, Ref.⁶³

To achieve a theoretical accuracy compatible with the expected precision of the MuSun experiment one must evaluate the μ^-d capture rate in HB χ PT at least to N²LO.^{47–50} An EFT*-based calculation of μ^-d capture was carried out by Ando *et al.*,⁴⁷ who used the value of d^R obtained in Ref.⁶³ by applying EFT* to tritium beta decay; Ando *et al.* report the value $\Gamma_d(\text{EFT}^*) = 386 \text{ s}^{-1}$. We remark en passant that, in deriving the so-called fixed terms of orders m_N^{-1} and m_N^{-2} , Ando *et al.*⁴⁷ used the Foldy-Wouthuysen transformation instead of the non-relativistic heavy baryon expansion. The two methods are not identical but one scheme can be transformed to the other as shown in, *e.g.*, Ref.⁸³ To order m_N^{-2} , the results of the two methods are identical. Note that in this EFT* calculation of Γ_d the weak transition operators are derived in HB χ PT whereas the two-nucleon wave functions are obtained using the Argonne *v*18 potential.⁷⁸ The high-momentum components of this NN potential is regulated by a Gaussian cut-off function. The inclusion of such regularization can in principle cause the violation of CVC (the conservation of the vector current) and PCAC (partial conservation of the axial current). Furthermore, the value of the LEC, d^R , becomes dependent on this regularization procedure, a topic which is also discussed in Ref.⁸⁴ However, if the numerical results for the observable Γ_d turns out to be stable against changes in the cutoff parameter, it is reasonable to conclude that, despite the above-mentioned formal issues, an EFT* calculation of Γ_d is practically model-independent.

The most detailed study to date of the μ^-d capture rate was made by Marcucci *et al.*,^{48,49} who carried out calculations based on both SNPA and HB χ PT. Their work also includes the calculation of the $\mu^3\text{He}$ capture reaction. In their SNPA calculation, Marcucci *et al.* used the initial and final nuclear wave functions for the $A = 2$ and 3 derived from the Argonne *v*18 two-nucleon potential⁷⁸ (in combination with the Urbana IX three-nucleon potential⁷⁹ in the case of $A = 3$). The relevant weak-interaction transition operators were obtained using SNPA,^{48,49} which involves one parameter, the N - Δ axial coupling constant that controls the two-body axial exchange current. After fixing this coupling constant by reproducing Γ_β^t , Marcucci *et al.* obtained $\Gamma_d(\text{SNPA}) = 390.4 \sim 390.9 \text{ s}^{-1}$, the lower (higher) value corresponding to the use of $g_A = 1.2654$ ($g_A = 1.2695$). It is to be noted that, the dependence of the results on the adopted value of g_A is significantly reduced

because of the constraint that the experimental value of Γ_β^t be reproduced for each choice of g_A .

In their HB χ PT calculation, Marcucci *et al.*^{48,49} used nuclear wave functions generated by the chiral N³LO two-nucleon potential,⁷¹ supplemented with the chiral N²LO three-nucleon potential⁶⁸ in the case of $A = 3$. The transition operators were derived to N³LO, which included two-pion exchange currents. To this order the theory still contains only one unknown LEC, d^R . This LEC was determined by reproducing Γ_β^t . In the spirit of low-energy EFT, Fourier transformation from momentum- to coordinate-space was regulated with a Gaussian regulator with a cutoff Λ , which was taken to be $\Lambda = 500 - 800$ MeV, following Park *et al.*⁶³ As mentioned, the stability of the results against the change of Λ is considered to give a measure of model-independence. Marcucci *et al.*^{48,49} obtained $\Gamma_d(\text{EFT}) = 393.6(7) \text{ s}^{-1}$ with practically no Λ -dependence within the range $\Lambda = 500 - 800$ MeV. Combining the results of their SNPA and HB χ PT calculations, Marcucci *et al.* concluded that the model dependence due to interactions, currents, and the cutoff Λ is at the 1 % level, and they gave as the best estimate the value $\Gamma_d = (389.7 - 394.3) \text{ s}^{-1}$.

At this order, like in the case of μ^-p capture, the radiative corrections need to be carefully studied. The HB χ PT-based evaluation of radiative corrections for μ^-d capture is yet to be completed.⁸⁵

7. Discussion and Summary

A topic closely related to muon capture on hydrogen is that of muon capture on ^3He . A measurement of this capture rate gave $\Gamma(\mu^3\text{He}) = 1496 \text{ s}^{-1}$ with 0.3% precision.⁴ An EFT* calculation of $\mu^3\text{He}$ capture was carried out by Gazit *et al.*,^{68,86} who used the Argonne $v18$ NN interactions⁷⁸ and the Urbana IX three-nucleon potential.⁷⁹ Most recently, Marcucci *et al.*^{48,49} evaluated $\Gamma(\mu^3\text{He})$ in both SNPA and HB χ PT, and they found good agreement between the SNPA and HB χ PT results, similarly to the case of μd capture. Marcucci *et al.* reported $\Gamma(\mu^3\text{He}) = 1494 \pm 21 \text{ s}^{-1}$. Radiative corrections obtained in the Marciano-Sirlin method²³ were used in arriving at this value. Agreement between theory and experiment is very satisfactory.

The μ^-p capture reaction, $\mu^- + p \rightarrow \nu_\mu + n$, discussed in Sec. 4 is often called ordinary muon capture (OMC) in contradistinction to radiative muon capture (RMC), $\mu^- + p \rightarrow \nu_\mu + n + \gamma$. It is noteworthy that the study of RMC in principle allows the determination of the q^2 dependence of $G_P(q^2)$ appearing in Eq.(5). For an obvious reason, however, RMC has a much smaller branching ratio than OMC, and for a longtime it was a great experimental challenge to observe RMC. Wright *et al.*⁸⁷ succeeded in measuring the highly suppressed RMC rate. However, the $G_P(q^2)$ extracted by Wright *et al.* is larger than what was derived from other experiments.^{5,6} Furthermore, the measured RMC capture rate does not agree with the theoretical value obtained in HB χ PT.^{25,88,89} This RMC experimental result remains a puzzle; see the discussions in the reviews^{5,6} for more details.

12 *Saori Pastore, Kuniharu Kubodera, Fred Myhrer*

The high-precision measurement of the hyperfine-singlet μ^-p capture rate Γ_0 by the MuSun Group has been conducive to intensive theoretical studies of this reaction based on HB χ PT. Recent developments include the HB χ PT calculation of the radiative corrections by Raha *et al.*,²⁷ and Pastore *et al.*'s work⁴⁶ on the propagation of uncertainties in the empirical values of the coupling constants, g_A and $g_{\pi NN}$ to uncertainties in the calculated value of Γ_0 . Pastore *et al.*⁴⁶ report $\Gamma_0 = 718 \pm 7 \text{ s}^{-1}$, which is in good agreement with the experimental value given in Eq.(1).

As for μ^-d capture, Marcucci *et al.*'s HB χ PT calculation of Γ_d is reported to have 1% accuracy, which matches the experimental accuracy of 1.5 % expected in the on-going MuSun measurements. Marcucci *et al.*^{48,49} used the radiative corrections calculated in the Sirlin-Marciano approach.²³ It is desirable to derive these radiative corrections within the HB χ PT framework. Such a calculation is currently underway.⁸⁵

Acknowledgements

This work is supported in part by the National Science Foundation, Grant No. PHY-1068305.

References

1. H. Primakoff, in *Nuclear and Particle Physics at Intermediate Energies* ed. J.B. Warren (Plenum, New York, 1975) p.1.
2. V. A. Andreev *et al.* (MuCap Collaboration), *Phys. Rev. Lett.* **110** (2013) 012504.
3. V. A. Andreev *et al.* [MuSun Collaboration], arXiv:1004.1754 [nucl-ex].
4. R. Ackerbauer *et al.*, *Phys. Lett. B* **417** (1998) 224.
5. P. Kammel and K. Kubodera *Annu. Rev. Nucl. Part. Sci.* **60** (2010) 327.
6. T. Gorringer and H. W. Fearing *Rev. Mod. Phys.* **76** (2004) 31.
7. L.E. Marcucci, *Int. J. Mod. Phys. A* **27** (2012) 1230006 [arXiv:1112.0113].
8. P. Kammel and L. E. Marcucci, in preparation.
9. H. Georgi, *Weak Interactions and Modern Particle Theory*, Addison-Wesley Publ. Comp. (NY, 1984), p. 80.
10. J. Gasser and H. Leutwyler, *Phys. Rep.* **87** (1982) 77; *Ann. Phys.* **158** (1984) 142.
11. J. Bijnens, *Prog. Part. Nucl. Phys.* **58** (2007) 521.
12. V. Bernard, N. Kaiser and U.-G. Meissner, *Int. J. Mod. Phys. E* **4** (1995) 193.
13. V. Bernard, *Prog. Nucl. Part. Phys.* **60** (2008) 82.
14. S. Scherer, *Prog. Nucl. Part. Phys.* **64** (2010) 1.
15. J. Beringer *et al.* (Particle Data Group) *Phys. Rev.* **86** (2012) 010001.
16. S. Weinberg, *Phys. Rev.* **112** (1958) 1375.
17. V. Bernard, N. Kaiser and U.-G. Meissner, *Phys. Rev. D* **50** (1994) 6899.
18. H. W. Fearing, R. Lewis, N. Mobed and S. Scherer, *Phys. Rev. D* **56** (1997) 1783.
19. S. L. Adler and Y. Dothan, *Phys. Rev.* **151** (1966) 1267.
20. L. Wolfenstein, in *High-Energy Physics and Nuclear Structure* ed. S. Devons (Plenum, New York, 1970) p. 661.
21. N. Kaiser, *Phys. Rev. C* **67** (2003) 027002.
22. A. Liesenfeld *et al.* *Phys. Lett. B* **468** (1999) 20.

23. A. Czarnecki, W. J. Marciano and A. Sirlin, *Phys. Rev. Lett.* **99** (2007) 032003.
24. S.-I. Ando, F. Myhrer and K. Kubodera, *Phys. Rev. C* **63** (2000) 015203.
25. V. Bernard, T. R. Hemmert and U.-G. Meissner, *Nucl. Phys. A* **686** (2001) 290.
26. G.I. Opat, *Phys. Rev.* **134** (1964) B428.
27. U. Raha, F. Myhrer and K. Kubodera, *Phys. Rev. C* **87** (2013) 055501.
28. D. Eiras and J. Soto, *Phys. Lett. B*, **491** (2000) 101.
29. J.L. Friar, *Ann. Phys. (NY)*, **122** (1979) 151.
30. A. Sirlin, *Phys. Rev.* **164** (1967) 1767.
31. W.J. Marciano and A. Sirlin, *Phys. Rev. Lett.*, **56** (1986) 22.
32. S. Ando, H. W. Fearing, V. Gudkov, K. Kubodera, F. Myhrer, S. Nakamura and T. Sato, *Phys. Lett. B* **595** (2004) 250.
33. D. Mund *et al.*, *Phys. Rev. Lett.* **110** (2013) 172502.
34. M.P. Mendenhall *et al.* (UCNA Collaboration), *Phys. Rev. C* **87** (2013) 032501.
35. F.E. Wietfeldt and G.L. Greene, *Rev. Mod. Phys.* **83** (2011) 1173.
36. A.N. Ivanov *et al.*, arXiv:1306.1995 (2013).
37. A.T. Yue *et al.*, *Phys. Rev. Lett.* **111** (2013) 222501.
38. T.E.O. Ericson, B. Loiseau and A.W. Thomas, *Phys. Rev. C* **66** (2002) 014005.
39. R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, R.L. Workman and M.M. Pavan, *Phys. Rev. C* **69** (2004) 035213 [arXiv:nucl-th/0311089].
40. D.V. Bugg, *Eur. Phys. J. C* **33** (2004) 505.
41. R.A. Arndt, W.J. Briscoe, I.I. Strakovsky and R.L. Workman, *Phys. Rev. C* **74** (2006) 045205 [arXiv:nucl-th/0605082].
42. V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga and D. R. Phillips, *Phys. Lett. B* **694** (2011) 473.
43. V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga and D. R. Phillips, *Nucl. Phys. A* **872** (2011) 69.
44. J.J. de Swart, M.C.M. Rentmeester and R.G.E Timmermans, *PiN Newslett.* **13** (1997) 96 [arXiv:nucl-th/9802084].
45. G. Höhler, in *Landolt – Börnstein* **9**, **b2** ed. H. Schopper (Springer, Berlin 1983).
46. S. Pastore, F. Myhrer and K. Kubodera, *Phys. Rev. C* **88** (2013) 058501.
47. S. Ando, T. S. Park, K. Kubodera and F. Myhrer, *Phys. Lett. B* **533**, 25 (2002) [nucl-th/0109053].
48. L. E. Marcucci, A. Kievsky, S. Rosati, R. Schiavilla and M. Viviani, *Phys. Rev. Lett.* **108** (2012) 052502 [arXiv:1109.5563 [nucl-th]].
49. L. E. Marcucci, M. Piarulli, M. Viviani, L. Girlanda, A. Kievsky, S. Rosati, R. Schiavilla, *Phys. Rev. C* **83** (2011) 014002.
50. J. Adam, Jr., M. Tater, E. Truhlik, E. Epelbaum, R. Machleidt and P. Ricci, *Phys. Lett. B* **709** (2012) 93 [arXiv:1110.3183 [nucl-th]].
51. S. Weinberg, *Physica A* **96** (1979) 327.
52. S. Weinberg, *Phys. Lett. B* **251** (1990) 288.
53. S. Weinberg, *Nucl. Phys. B* **363** (1991) 3.
54. S. Weinberg, *Phys. Lett. B* **295** (1992) 114.
55. T.-S. Park, D.-P. Min, M. Rho, *Nucl.Phys. A* **596** (1996) 515-552.
56. T.-S. Park, D.-P. Min, M. Rho, *Phys.Rept.* **233** (1993) 341-395.
57. S. Kölling, E. Epelbaum, H. Krebs, U.-G. Meissner, *Phys. Rev. C* **80**, 045502 (2009).
58. S. Kölling, E. Epelbaum, H. Krebs, and U.-G. Meissner, *Phys. Rev. C* **84**, 054008 (2011).
59. S. Pastore, R. Schiavilla, and J.L. Goity, *Phys. Rev. C* **78**, 064002 (2008).
60. S. Pastore, L. Girlanda, R. Schiavilla, M. Viviani, and R. B. Wiringa, *Phys. Rev. C* **80**, 034004 (2009).

14 *Saori Pastore, Kuniharu Kubodera, Fred Myhrer*

61. M. Piarulli, L. Girlanda, L.E. Marcucci, S. Pastore, R. Schiavilla, and M. Viviani, *Phys. Rev. C* **87**, 014006 (2013).
62. A. Baroni *et al.*, in preparation.
63. T. S. Park *et al.*, *Phys. Rev. C* **67** (2003) 055206 [arXiv:nucl-th/0208055].
64. L.E. Marcucci, R. Schiavilla and M. Viviani, *Phys. Rev. Lett.* **110** (2013) 192503.
65. S. Nakamura, T. Sato, S. Ando, T. S. Park, F. Myhrer, V. P. Gudkov and K. Kubodera, *Nucl. Phys. A* **707** (2002) 561 [nucl-th/0201062].
66. C. Hanhart, U. van Kolck and G.A. Miller, *Phys. Rev. Lett.* **85** (2000) 2905.
67. V. Baru, C. Hanhart and F. Myhrer, *Int. J. Mod. Phys.* **23** (2014) 1430004 [arXiv:1310.3505].
68. D. Gazit, S. Quaglioni and P. Navratil, *Phys. Rev. Lett.* **103** (2009) 102502 [arXiv:0812.4444 [nucl-th]].
69. E. Epelbaum, A. Nogga, W. Gloeckle, H. Kamada, U.-G. Meißner and H. Witala, *Phys. Rev. C* **66** (2002) 064001 [arXiv:nucl-th/0208023].
70. E. Epelbaum, H. -W. Hammer and U. -G. Meissner, *Rev. Mod. Phys.* **81** (2009) 1773 [arXiv:0811.1338 [nucl-th]].
71. R. Machleidt and D. R. Entem, *Phys. Rept.* **503** (2011) 1 [arXiv:1105.2919 [nucl-th]].
72. V. Lensky, V. Baru, J. Haidenbauer, C. Hanhart, A. E. Kudryavtsev and U. G. Meissner, *Eur. Phys. J. A* **26**, 107 (2005) [arXiv:nucl-th/0505039].
73. V. Lensky, V. Baru, E. Epelbaum, C. Hanhart, J. Haidenbauer, A. E. Kudryavtsev and U. G. Meissner, *Eur. Phys. J. A* **33**, 339 (2007) [arXiv:0704.0443 [nucl-th]].
74. A. Gardestig, *Phys. Rev. C* **74** (2006) 017001 [arXiv:nucl-th/0604035].
75. A. Gardestig and D. R. Phillips, *Phys. Rev. Lett.* **96** (2006) 232301. [arXiv:nucl-th/0603045].
76. A. Gardestig and D. R. Phillips, *Phys. Rev. C* **73** (2006) 014002 [nucl-th/0501049].
77. A. Gardestig, *J. Phys. G* **36** (2009) 053001 [arXiv:0904.2787 [nucl-th]].
78. R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, *Phys. Rev. C* **51** (1995) 38.
79. B.S. Pudliner *et al.*, *Phys. Rev. C* **56** (1997) 1720.
80. J. Carlson and R. Schiavilla, *Rev. Mod. Phys.* **70** (1998) 743.
81. N. Tataru, Y. Kohyama and K. Kubodera, *Phys. Rev. C* **42** (1990) 1694.
82. J. Adam, E. Truhlik, S. Ciechanowicz and K.-M. Schmitt, *Nucl. Phys. A* **507** (1990) 675.
83. A. Gårdestig, K. Kubodera and F. Myhrer, *Phys. Rev. C* **76** (2007) 014005.
84. P. Ricci, E. Truhlik, B. Mosconi, J. Smejkal, *Nucl. Phys. A* **837** (2010) 110.
85. Y.-H. Song *et al.*, in preparation.
86. D. Gazit, *Phys. Lett. B* **666** (2008) 472.
87. D. H. Wright *et al.* *Phys. Rev. C* **57** (1998) 373.
88. T. Meissner, F. Myhrer and K. Kubodera, *Phys. Lett. B* **416** (1998) 36.
89. S. Ando and D.-P. Min, *Phys. Lett. B* **417** (1998) 177.